

# Matching in Multi-Agent Pathfinding using $M^*$

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## Abstract

Multi-agent pathfinding (*MAPF*) is the process of finding collision-free paths for multiple agents. *MAPF* can be extended by grouping agents into teams. In a team, agents need to be assigned (or matched) to one of the team’s goals such that the *sum of individual costs* is minimised. This extension is called *MAPF* with matching (*MAPFM*).  $M^*$  is a complete and optimal algorithm to solve *MAPF* problems. In this paper, two strategies are proposed which allow  $M^*$  to solve *MAPFM* problems. These strategies are called *inmatching* and *prematching*. It is shown that *prematching* is generally preferable to *inmatching*, the benefits of different optimisations for  $M^*$  are compared, and it is shown that the performance of  $M^*$  performs very comparably to other  $A^*$ -derived algorithms.

## 1. INTRODUCTION

A large number of real-world situations require the planning of collision-free routes for multiple agents.<sup>1</sup> For example, the routing of trains over a rail network [1], directing robots in warehouses [2] or making sure autonomous cars do not collide on the road [3]. Problems of this nature are called *Multi-agent pathfinding* problems, which will hereafter be abbreviated to *MAPF*.

In *MAPF*, each agent has a starting position and a goal position. For every agent, a route needs to be found from their start to their goal, without two agents colliding. Finding these collision-free routes has been formally proven to be *NP-hard* [4].

One algorithm to solve *MAPF* is called  $M^*$  [5].  $M^*$  is derived from the  $A^*$  algorithm [6] as described by Standley [7].  $A^*$  generally plans the paths of agents to-

gether. This means that in each timestep, the number of possible next states grows exponentially with the number of agents. Contrasting that, in  $M^*$ , agents follow an individually optimal path whenever possible. In each timestep, only the subset of colliding agents is jointly planned and if necessary, backtracking is performed.

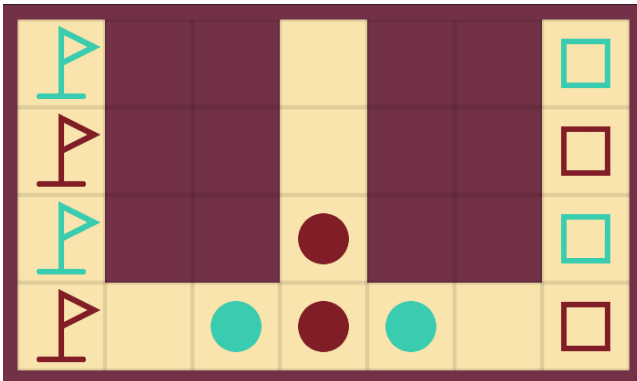


Figure 1: A snapshot of agents solving a *MAPFM* problem. Two teams of agents are moving from their starts to their goals. Squares, flags and circles are starts, goals and agents respectively. One red agent (at four squares from the left, three squares from the top) waits to let the three other agents pass.

When directing robots through warehouses, there may be different models of robots with different capabilities. Some robots may be able to restock shelves while others collect orders. The *MAPF* problem can be generalised to represent such a problem with teams of agents. In contrast to *MAPF*, an agent does not have a single goal in this generalisation. Instead, agents can use any goal associated with the team it is in, but no two agents can end at the same goal. An algorithm solving this problem must assign each agent to a goal. Such an assignment is called a matching. This problem is therefore named multi-agent pathfinding with matching (hereafter abbreviated to *MAPFM*).

In this paper, *MAPFM* is first defined, after which two

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<sup>1</sup>An agent is a collective name for a single unit for which a path is calculated. The word ‘agent’ could refer to a single robot, a train or a car, but can also refer to an abstract or simulated entity. In *MAPF*, paths are planned for multiple agents.

techniques are discussed which extend  $M^*$  to also solve *MAPFM* problems. These two techniques are then compared, to each other, and to a number of other algorithms which also solve *MAPFM* problems. Apart from this comparison, a number of extensions to  $M^*$  are discussed which improve  $M^*$ 's performance. Some of these extensions can also improve the performance of  $M^*$  on plain *MAPF* problems.

### 1.1. PRIOR WORK

Before this research, a separate study has been performed [8] on a problem called the target-assignment and pathfinding (*TAPF*) problem. The difference between *TAPF* and *MAPFM* is that in *TAPF*, the *makespan* (the cost of the longest path) instead of the *sum of individual costs* (the cost of all paths combined) is minimised. To solve *TAPF*, conflict based search (*CBS*) [9] is used by [8], using a min-cost flow-based algorithm for the lower level. The min-cost flow is used to solve *TAPF* for a single team in polynomial time. The higher level of *CBS* combines single team solutions to find a solution for all teams. According to [10], such a Min-cost flow-based algorithm can also be used to solve *MAPFM* problems.

To the best of the author's knowledge, solving *TAPF* or *MAPFM* with other algorithms than *CBS* (such as  $M^*$ ) has not yet been explored.

In contrast, a lot of research has been done finding and improving algorithms that solve *MAPF* [5, 7, 9, 11–14]. The current state-of-the-art *MAPF* algorithms are *Conflict Based Search (CBS)* and *Branch-and-Cut-and-Price (BCP)*. However, which of the existing algorithms for *MAPF* can be adapted such that they also perform well at solving *MAPFM* problems is yet unclear.

## 2. DEFINITIONS OF *MAPF* AND *MAPFM*

The definition of multi-agent pathfinding used in this paper is based on Stern's definition given in [15]. The definition is given below.

A *MAPF* problem  $P$  consists of the following elements:

$$P = \langle G, s, g \rangle$$

- $G$  is an undirected graph  $\langle V, E \rangle$ 
  - $V$  is a set of vertices
  - $E$  is a set of unweighted edges between vertices
- $s$  is a list of  $k$  vertices where every  $s_i$  is a starting position for an agent  $a_i$
- $g$  is a list of  $k$  vertices where every  $g_i$  is a goal position for an agent  $a_i$

The goal of *MAPF* is to find a path for all agents  $a_i$  from  $s_i$  to  $g_i$ , without vertex and edge conflicts. This means that in a timestep  $t$ , two agents may not be on the same vertex, and between two timesteps, two agents may not travel over the same edge.

The cost  $c_i$  of a path  $\pi_i$  is the number of timesteps until

the last time that agent  $a_i$  arrives at its goal  $g_i$ . Two primary approaches to define the cost  $c$  of a solution to a *MAPF* instance are discussed in [15]. The *sum of individual costs* is the sum of the cost of all paths ( $c = \sum_{n=0}^k c_n$ ). Alternatively, the *makespan* is equal to the maximum cost of all paths ( $c = \max_{n=0}^k c_n$ ). An optimal solution to a *MAPF* instance is a set of paths  $\pi$  with the smallest possible cost  $c$ .

Although the algorithms presented in this paper would work on any graph  $G$ , in examples and experiments,  $G$  is simplified to a 4-connected grid. In this paper, this grid is sometimes called a 'map', on which agents move from their start positions to their goal positions.

### *MAPFM*

Now, the concept of matching is added to the definition of *MAPF*, to create multi-agent pathfinding with matching (*MAPFM*). The new *MAPFM* problem adds two elements to  $P$ , as shown below:

$$P' = \langle G, s, g, sc, gc \rangle$$

- $sc$  is a list of colours. Each starting vertex  $s_i$  is assigned a colour  $sc_i$ .
- $gc$  is a list of colours. Each starting vertex  $s_i$  is assigned a colour  $sc_i$ .

This definition divides all agents into teams. An agent  $a_i$ 's team colour is the colour of its starting location  $sc_i$ . In total there are  $K$  teams  $k_1 \dots k_n$ .  $K$  is equal to the number of different colours in  $sc$  and  $gc$ .

In *MAPFM*, agents do not have a single goal. Instead, teams have several goals which all have the same colour  $gc_i$ . A team has as many goals as there are agents. A solution to *MAPFM* is a set of collision-free paths for each agent, in which every agent travels to a goal with a colour equal to the colour of that agent.

*MAPFM* uses the *sum of individual costs* as an optimisation criterion.

## 3. A DESCRIPTION OF $M^*$

$M^*$  [5] is a complete and optimal pathfinding algorithm, similar to  $A^*$ . However, unlike  $A^*$ ,  $M^*$  is specifically designed to solve *MAPF* problems. To use  $A^*$  for *MAPF*, the planning of paths for all agents needs to be coupled to make sure no states are expanded in which collisions occur. Coupled planning means that the states used in  $A^*$  contain the positions of all agents at the same time, and every expansion expands new states for all agents at the same time. The search space is said to be  $k$ -dimensional where  $k$  is the number of agents for which paths are planned.

Unlike in  $A^*$ , in  $M^*$  the planning of agents is initially not coupled. Instead,  $M^*$  plans agents according to an individually optimal path. An individually optimal path is the minimal cost path for an agent to their goal, not taking into account the location and movement of other agents. This may therefore lead to the collision of agents. However, as long as collisions do not occur,

this strategy separates the planning of agents, making the search space one-dimensional.

However, with many map layouts, collisions between agents following individually optimal paths are bound to occur. When  $M^*$  detects the creation of a state  $s$  in which two or more agents do collide, this state is discarded. Instead, parent states of  $s$  are reassessed to see if the colliding agents can plan differently, in a way that may not be individually optimal but avoids the collision in  $s$ .

During this reassessment, the planning of only the agents which collided in  $s$  is coupled like in  $A^*$ . The search space of  $M^*$  can therefore be thought of as being one-dimensional, but with a dynamically growing dimensionality around collisions. Around collisions, the search space is  $n$ -dimensional where  $n$  is the number of colliding agents.

Just like  $A^*$ ,  $M^*$  uses a priority queue and a heuristic to prioritise the exploration of states. This finds optimal solutions as long as an admissible heuristic is used such as the Euclidean or Manhattan distance to goal location of each agent.

#### 4. $M^*$ AND MATCHING

To add matching to  $M^*$ , this paper proposes two options which are called "inmatching" and "prematching". In this section, both are explained and their advantages and disadvantages are discussed.

##### 4.1. INMATCHING

*Inmatching* is the process of performing matching as a part of the pathfinding algorithm that is used. To understand the working of *inmatching*, it is useful to first look at *inmatching* in the much simpler  $A^*$  algorithm. With  $A^*$ , the expansion of a state consists of all possible moves for all agents.  $A^*$  searches through the search space until the goal state has been removed from the frontier. With an admissible heuristic,  $A^*$  guarantees that backtracking from this first goal state gives a shortest path between the start and goal location.

With *inmatching*, there is not one goal state. Instead, any state in which all agents stand on a goal of their own team is considered a goal state. This means there is more than one goal state. An admissible heuristic for *inmatching*  $M^*$  is the Euclidean or Manhattan distance to the closest goal location for each agent.

##### Inmatching applied to $M^*$

*Inmatching* can similarly be used with  $M^*$ . However, in  $M^*$  *inmatching* is likely to be quite inefficient (an experimental evaluation of this inefficiency can be found in Section 7.1). The reason for this inefficiency is that in  $M^*$ , agents try to follow an individually optimal path. In *MAPF* problems, an agent only has one individu-

ally optimal path because an agent only has one goal position. However, in *MAPFM*, agents commonly have multiple goal positions. With *inmatching*, the paths to all goal positions of an agent need to be considered to guarantee optimality. In a sense, an agent has multiple individually optimal paths. One towards each goal position of the agent's team.

In regular  $M^*$  the branching factor <sup>2</sup> is one when agents do not collide. However, because with *inmatching* there are multiple individually optimal paths, the branching factor of  $M^*$  may be larger than one even when there are no collisions.

The upper bound of this branching factor can be expressed as follows:

$$\prod_{i=1}^k \text{goals}(a_i)$$

Here, the *goals* function gives the number of goals in the team of agent  $a_i$ . All agents need to consider moving towards all goals of the team they are in. A new state needs to be created for every combination of movements of agents because a single state contains the location of all agents combined. Therefore the upper bound for the number of expanded states is a product of the movements for a single agent.

*inmatching*  $M^*$  will hereafter be abbreviated to *imM\**

##### 4.2. PREMATCHING

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###### Algorithm 1: prematch $M^*$

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**Result:** Find the matching  $m_{best}$  with the smallest cost

$m_{best} \leftarrow \emptyset;$

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$matchings \leftarrow \text{find all matchings}(starts, goals);$

**foreach**  $m \in matchings$  **do**

$s \leftarrow \text{mstar}(starts, m)$  {Evaluate with  $M^*$ };  
     $m_{best} \leftarrow \text{min}(m_{best}, s);$

**end**

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As an alternative to *inmatching*, there is *prematching*. With *prematching*, the *MAPFM* problem is transformed into a number of *MAPF* problems. Each possible matching is calculated in advance, and normal  $M^*$  as described by [5] is performed on each matching exhaustively. Algorithm 1 shows this exhaustive search.

In Section 5.3 an improvement to *prematching*  $M^*$  is proposed which uses a heuristic to prune some of the matchings which otherwise needed to be evaluated.

*prematching*  $M^*$  will hereafter be abbreviated to *pmM\**

<sup>2</sup>The branching factor is the number of child states which are created from one parent state. A branching factor of one means that each parent only creates one child state. In this text, a state refers to an  $M^*$  state as described in Section 3.

## 5. EXTENSIONS TO $M^*$

$M^*$  can be improved upon in various ways. Some of these extensions are only applicable to  $M^*$  with matching, while others improve  $M^*$  itself. In this section, these extensions are described in detail.

### 5.1. PRECOMPUTED PATHS AND HEURISTICS

A part of solving  $M^*$  instances is finding the individually optimal path for each agent. This can be done with conventional pathfinding algorithms such as  $A^*$ . But to avoid repeated calculation, it is important to first create a lookup table of the minimal distance to each goal, for every open square on the map. This table can be created using a breadth-first search from every goal. With this table, finding in which way an agent should move to reach the goal in an individually optimal manner becomes a constant time operation.

When using *prematching*, the same goal locations and start locations are (although in a different permutation) reused multiple times. The lookup table can remain the same for all evaluations of matchings.

Using this lookup table, a better heuristic for  $M^*$  can be created. Instead of using the Euclidean or Manhattan to the goal, the lookup table can be used to find the exact distance, taking the obstacles on the map into account.

### 5.2. OPERATOR DECOMPOSITION

In [7], Standley describes a technique called operator decomposition (*OD*). Operator decomposition is designed to be an extension to  $A^*$  that improves the runtime of solving *MAPF* problems.

In *MAPF*, when an agent moves, it can perform one of five actions (assuming a grid map is used): wait on the same square, or move in one of the four cardinal directions. With  $A^*$ , the movement of all agents is coupled. When a state is expanded, every child state contains the new position of every agent. This means that if there are 5 agents, which may all perform one of 5 possible actions,  $5^5 = 3125$  child states are created and added to the frontier. This is called a full expansion.

Operator decomposition can lower this large branching factor by introducing the concept of partial states. As the name implies, when a state is removed from the frontier, it is not fully expanded. Instead, only the 5 possible actions of a single agent are evaluated. These 5 partial states are added to the frontier. When a partial state is removed from the frontier, the actions of the next agent are evaluated too until the actions of all agents are evaluated and a full expansion is reached again.

So in the worst case, *OD* still performs a full expansion. However, the advantage of *OD* is that partial states can also be prioritised in the frontier. When the expansion of a subset of agents is found to result in a low heuristic,

the partial state is given a higher priority to be further expanded. This may lead to finding solutions to *MAPF* instances more quickly.

Even though  $M^*$  tries to avoid the coupled planning of agents as much as possible, when collisions occur, branching factors can still become large. *OD* can help  $M^*$  to reduce this branching factor when solving *MAPF* problems [16]. In Section 7.2 the benefits *OD* has on  $M^*$  when solving *MAPFM* problems are evaluated.

### 5.3. PRUNING OF MATCHINGS

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**Algorithm 2:** prematch  $M^*$  with pruning

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**Result:** Find the matching  $m_{best}$  with smallest cost, pruning when possible.

$m_{best} \leftarrow \emptyset$

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$M \leftarrow$  find all matchings(*starts*, *goals*);  
sort( $M$ ) {on heuristic; ascending};

**foreach**  $m \leftarrow M$  **do**

**if** *heuristic*( $m$ ) < *cost*( $m_{best}$ ) **then**

$s \leftarrow$  mstar(*starts*,  $m$ );

$m_{best} \leftarrow$  min( $m_{best}$ ,  $s$ );

**end**

**end**

---

$pmM^*$ , as previously described, has to evaluate every matching. However, it is sometimes possible to discard some matchings without evaluating them with  $M^*$  at all, by using heuristics.

To do this, an admissible heuristic (e.g. the sum of distances between start and goal locations) of the initial state of every matching is calculated. This represents a lower bound for the cost of this matching.

Then, when *prematching* evaluates every matching  $m$ , it keeps track of the best matching  $m_{best}$  which is the matching with the lowest cost so far. However, when *heuristic*( $m$ )  $\geq$  *cost*( $m_{best}$ ),  $m$  can immediately be pruned, because this matching  $m$  can never yield a solution that has a lower cost than  $m_{best}$ .

#### Sorting

To take maximum advantage of pruning, matchings can be sorted based on their heuristic. Making sure the matching with the lowest heuristic is evaluated first can increase the chances that later matchings are pruned. Algorithm 2 shows how pruning with sorting works.

## 6. EXPERIMENTAL SETUP

To evaluate the performance of matching  $M^*$ , a number of experiments were performed. For each of these experiments, the algorithm used is written in Python 3.9 and benchmarks were run on a virtualized system with a 12 core Intel Xeon E5-2683 running at 2GHz, which has 8GiB of RAM.

Various factors are influencing how a *MAPFM* solving

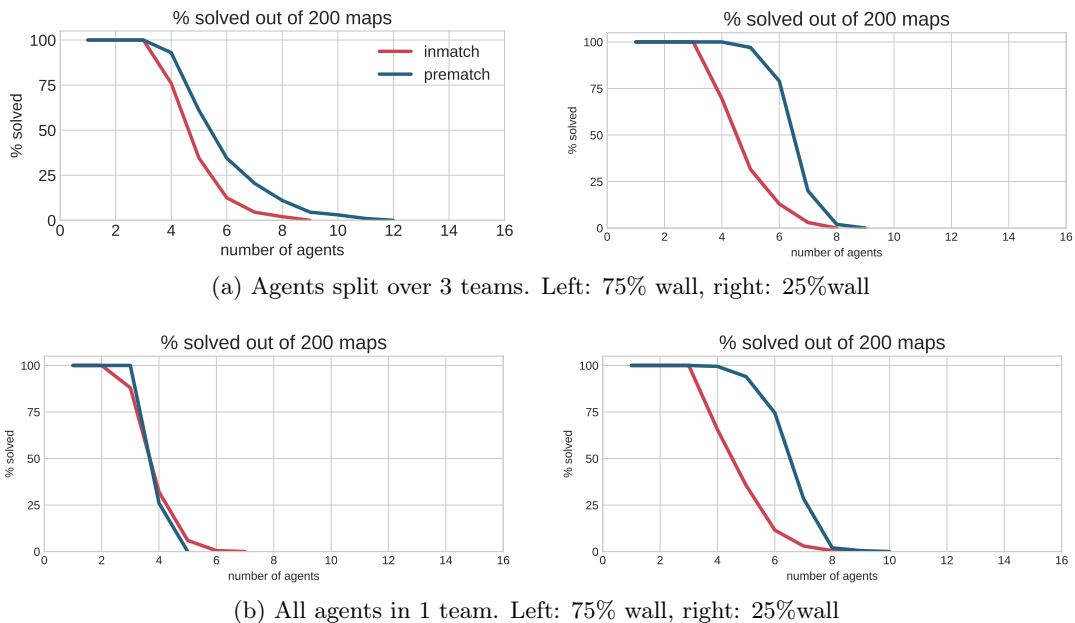


Figure 2: Percentage of maps solved in 2 minutes out of 200 random  $20 \times 20$  maps.

algorithm performs in benchmarks. Three important factors are the total number of agents, the number of teams over which the agents are distributed, and the layout of the maps on which benchmarks are run.

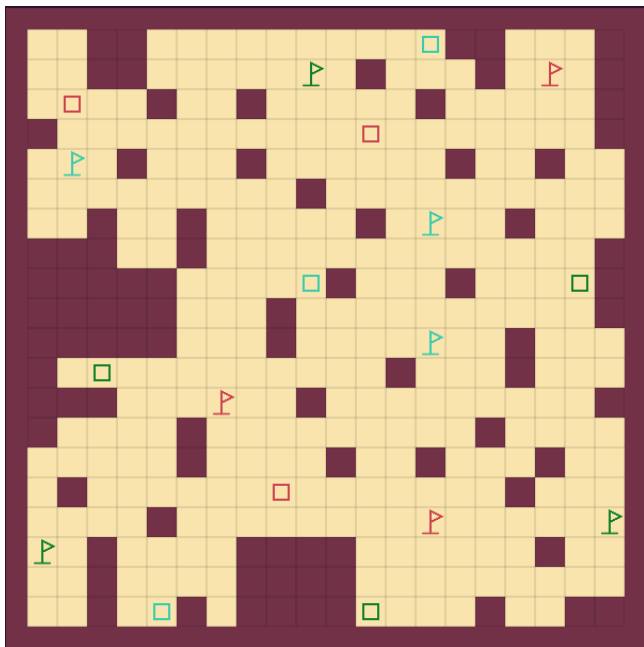


Figure 3: A map which is 25% obstacle, with agents distributed in three teams.

The experiments are grouped into sets of four *scenarios*, to show the differences in performance as these parameters vary. Performance is assessed on maps where either 25% or 75% of the grid is an obstacle, and in situations

where agents are grouped into either one or three teams.

For these benchmarks,  $20 \times 20$  grid maps<sup>3</sup> are randomly generated such that they are guaranteed to be solvable (a proof of which can be found in A). Four sets of maps were generated for each experiment with the previously discussed parameters. Agents and goals are uniformly distributed on this map and teams are assigned randomly. When the team size does not divide the number of agents, there will be one smaller team. An example of such a randomly generated map can be found in Figure 3.

Because *MAPFM* is an *NP-Hard* problem, it is possible that finding the solution to an instance of the problem takes an unreasonable amount of time. Therefore, in each experiment, a timeout of two minutes is used. Experimental results show the percentage of maps (out of a set of 200) which the algorithm manages to solve within this timeout.

## 7. RESULTS AND DISCUSSION

In this section, the results of a number of experiments are shown. The purpose of this is first to show which of the two matching strategies is superior, and why this is the case. Secondly, the experiments quantify the benefits of different extensions to *pmM\**. Lastly, the results of the experiments are put into context by comparing *M\** to other *MAPFM* algorithms performing the same experiments.

### 7.1. MATCHING STRATEGIES

In Section 4, two strategies were proposed for adding matching to *M\**. Both strategies were tested following

<sup>3</sup>Maps of different sizes will likely also show differences in performance. However, experiments with other map sizes are not included in this paper.

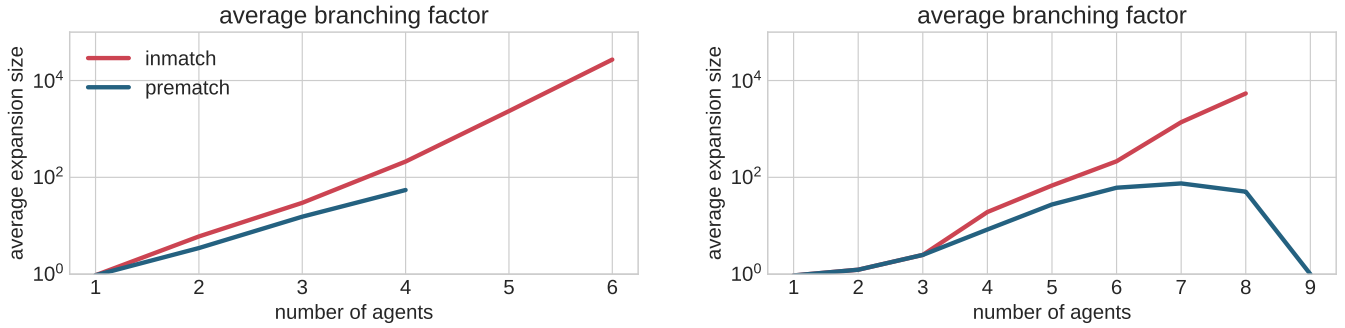


Figure 4: Average branching factor on the same maps as used in figure 2b (i.e. all agents in one team). Left: 75% wall, right: 25% wall. The blue line in the left figure stops earlier because *prematching* was not able to solve maps with more agents.

the experimental setup described in Section 6. In Figure 2 the results of this experiment are shown.

From these graphs, it can be seen that *prematching* is generally superior to *inmatching* in all but one of the scenarios. Only in the 75% benchmark of figure 2b *inmatching* outperforms *prematching* slightly.

### Branching factor

In Section 4.1, it was hypothesised that this difference in performance could occur, because of the larger branching factor.

To demonstrate this is indeed the case, another, separate, experiment was performed. Again on 200 random maps as described in Section 6, but this time showing the average number of states expanded each expansion. The exact same maps were used as in Figure 2b.

In Figure 4, the outcomes of this experiment are shown on a logarithmic scale. Indeed, on average, *inmatching* expands many more states.

### When *inmatching* is better

However, in the scenario where 75% of the map is an obstacle in Figure 2b *inmatching* does seem to perform slightly better than *prematching*. When agents are all in one team, there are more matchings compared to when agents are separated into multiple teams. With *prematching*, each matching needs to be exhaustively searched, which is slow.

In contrast, the problem of *inmatching* expanding so many states is reduced in this scenario. There are fewer directions for agents to go because of the large number of obstacles.

So in general, when there are many matchings, but few directions for agents to move in, *inmatching* may perform slightly better.

## 7.2. EXTENSIONS TO $M^*$

Various extensions have been proposed in Section 5. To show the benefits of each extension, they were compared following the experimental setup described in Section 6.

In Figure 5, the performance of these extensions is shown. Each line in the graph shows a new extension added to all of the previous extensions. For example, the line displaying operator decomposition also uses pre-computed heuristics and all other previous extensions.

Because in Section 7.1, *prematching* was observed to generally perform better, only the *prematching* matching strategy is used in this experiment.

### Pruning and sorting

In the experiments where 25% of the map is filled with obstacles, pruning and sorting have a large impact on performance. However, in graphs where 75% is a wall, there is barely a difference at all. The heuristic used to prune is the Manhattan distance between the agent start locations and the closest goal. This does not take into account the obstacles on the map.

Pruning is only possible when the heuristic is larger than the best matching found so far (as described in Section 5.3). This does not happen often with this inaccurate heuristic. Therefore, using the precomputed heuristic (also shown in Figure 5) makes a large difference.

When there are fewer obstacles in the map, the probability of pruning is much higher, explaining the difference in the performance of pruning and sorting between these two scenarios.

### Memory usage

The best variant of  $M^*$  can be seen solving maps with 14 agents in Figure 5a. After this, the percentage abruptly drops to zero. The reason for this is not that finding the solution takes too long. Instead, attempts were terminated by the operating system because too much memory was used. Attempts on maps with 15 agents saw memory usages of over 8GiB to solve a single instance.

## 7.3. COMPARISON WITH OTHER ALGORITHMS

This research is part of a set of parallel studies on how to extend a collection of *MAPF* algorithms with matching. All these studies used exactly the same problem definition but using one of the following base algorithms:

- Extended partial expansion  $A^*$  (*EPEA\**) [12]

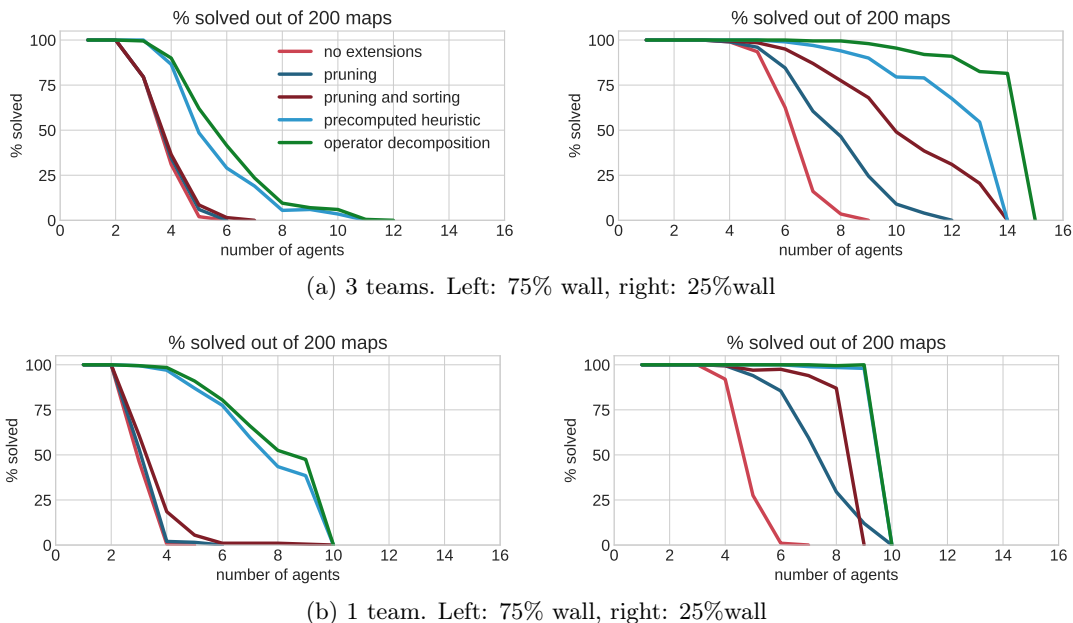


Figure 5: Percentage of maps solved in 2 minutes out of 200 random  $20 \times 20$  maps. Each line and extension to  $M^*$  is graphed in combination with all previous extensions. For example, the line displaying *operator decomposition* also uses precomputed heuristics and all other previous extensions.

- (implementation and research by [17])
- $A^*$  with operator decomposition and independence detection ( $A^*+OD+ID$ ) [7] (implementation and research by [18])
  - Increasing cost tree search (ICTS) [14] (implementation and research by [19])
  - Conflict-Based Min-Cost-Flow (CBM) [8] (implementation and research by [10])

Experiments were performed following the experimental setup described in Section 6. All algorithms were benchmarked on the same computer to ensure a fair comparison. Still, it is hard to compare algorithms well because, for example, *CBM* made use of external libraries written in *C++*. But, compared to the exponential nature of *MAPF*, such a strategy providing a linear speed-up is relatively insignificant. Figure 6 shows the results of this comparison.

In Figure 6b, it may look like an error was made. *CBM* solves 100% of the maps up to maps with 25 agents. In fact, *CBM* can solve maps with many more agents. *CBM* makes use of min-cost flow to solve *MAPFM* for single teams in polynomial time. Solutions for single teams are checked and modified when conflicts are found between agents in different teams.

This means that in cases where there *is* only one team, *CBM* can solve *MAPFM* in polynomial time, scaling linearly with the number of agents for which a route needs to be found.

In the experiments where multiple teams were used (as shown in Figure 6a), *CBM* performs more comparably with other algorithms. Still, when there are few obstacles, *CBM* excels.

Apart from *CBM*, all other algorithms (including  $M^*$ ) show very similar performance characteristics. All these algorithms use a form of prematching, where all matchings are exhaustively searched. This seems to be a factor that limits the capabilities of all these algorithms.

Only  $A^*+OD+ID$  performs exceptionally well on maps 25% filled with obstacles, and with three teams. The reason for this is likely due to the way independence detection works in the implementation described in [18].

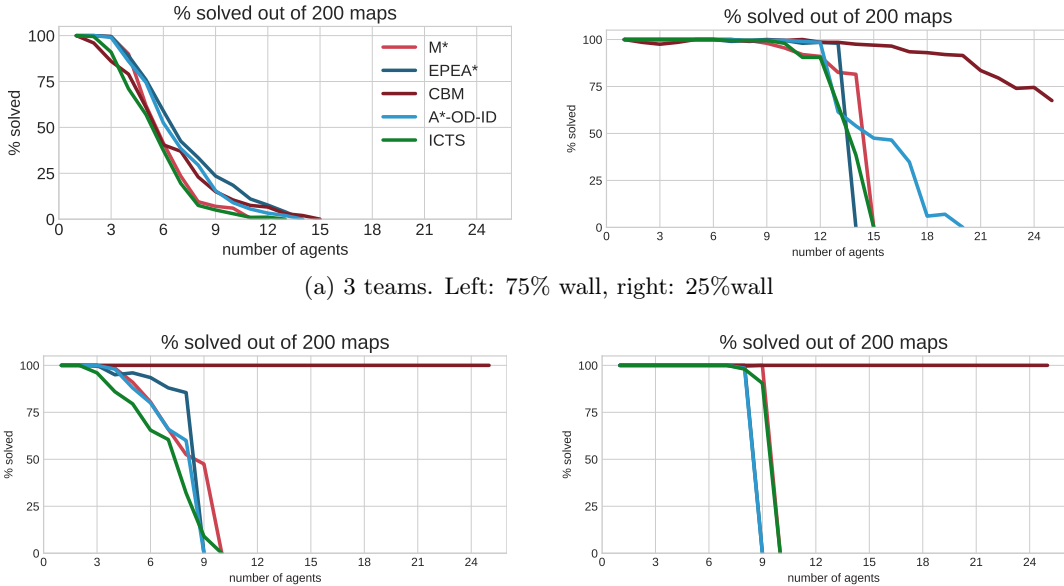
It must however be said, that this comparison is **not** complete. For example, testing with agents split over more teams, on larger maps, or with specifically crafted obstacles such as long corridors may show very different results.

## 8. CONCLUSION

In this paper, a generalisation of multi-agent pathfinding (*MAPF*) was introduced, called matching. This new problem is called *MAPFM*.  $M^*$ , an algorithm for *MAPF*, was modified to solve these *MAPFM* problems. To do this, two strategies were explored called *inmatching* and *prematching*. Experimental results showed that in many cases, *prematching* is superior to *inmatching*.

Subsequently, several improvements to *prematching*  $M^*$  were considered and their benefits were experimentally evaluated. Pruning and sorting of matchings, using precomputed heuristics, and independence detection were shown to have a large effect on performance.

Finally, it was shown that *pmM^\** has very similar performance characteristics as *ICTS*, *EPEA^\** and  $A^*+OD+ID$ . *CBM* performed notably better than  $M^*$



(a) 3 teams. Left: 75% wall, right: 25% wall

(b) 1 team. Left: 75% wall, right: 25% wall. In the right graph, the lines for  $EPEA^*$  and  $A^*+OD+ID$  overlap, and so do the lines for  $ICTS$  and  $M^*$ .

Figure 6: Percentage of maps solved in 2 minutes out of 200 random  $20 \times 20$  maps.

in experiments performed in this paper.

## 9. FUTURE WORK

### Partial Expansion

One problem with the *inmatching* strategy presented in Section 4.1 is that the branching factor is very large. However, a lot of the expanded states are never needed or accessed again. Even with *prematching* this was a problem. For example, while creating the graphs in Figure 5, the data points where 14 or 15 agents were involved could use as much as 8GiB of memory.

*Partial-* or *enhanced partial expansion* as presented in [12] and [13] only expands states when they are needed to reduce memory usage. It is likely that partial expansion can also be applied to  $M^*$  to improve its memory performance.

### Conflict based search

In this paper, matching  $M^*$  was compared to several other algorithms. One of those was CBS [9]. However, this implementation used a min-cost flow-based approach similar to the one described in [8], but altered to find the *sum of individual costs* instead of the *makespan*.

*CBS*, without matching and these min-cost flow extensions, is one of the most studied algorithms for *MAPF* ([9, 20–23]). Comparing  $M^*$  with *inmatching* and *prematching* to *CBS* using *inmatching* and *prematching* as well could provide useful data.

### Waypoints

In previous research [24–27], *MAPF* was extended with waypoints instead of matching. It may be possible to

combine these two extensions. For example, each team may have a set of waypoints. It does not matter which agent visits which waypoint as long as all waypoints are visited. Alternatively, each agent has its own waypoints, but the goals are still shared with a team of other agents.

### Agents and goals

In the definition of matching presented in this paper, in each team the number of goals was always equal to the number of agents. However, in real-world scenarios this may not always be the case.

For example, there may be a number of robots who have a number of tasks to do. Some robots cannot perform certain tasks so robots are teamed. But there may be more tasks than robots in each team, and robots will need to prioritise. Some research has already been done in this area, but this uses *TAPF* as a basis [28].

### Recursive $M^*$

In [5], an extension to  $M^*$  is described called recursive  $M^*$ . It could increase the performance of  $M^*$  on regular *MAPF* problems. Whether recursive  $M^*$  makes a difference combined with matching, has not yet been verified.

## 10. REPRODUCIBILITY

Results in this paper have been generated using an implementation of  $M^*$  made in Python specifically for this research. The code for this, together with the maps and raw experimental results, are publicly available on Github at <https://github.com/jonay2000/research-project>, doubly licensed under the Apache 2.0 and MIT licenses. Reports of bugs and new additions to this repository are always welcome.



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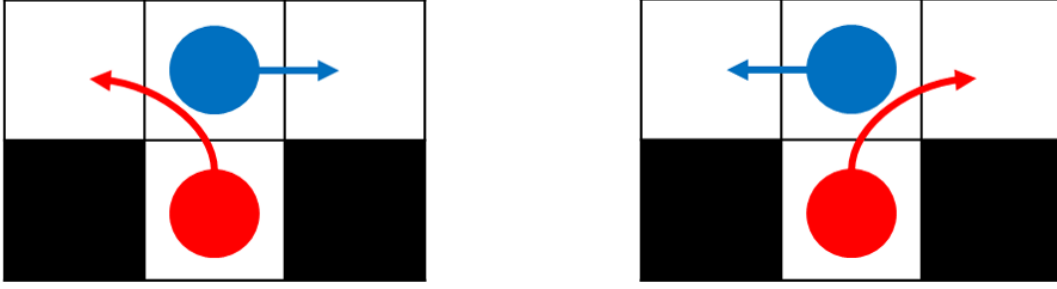


Figure 7: Agents passing each other on branch vertices

## A. GENERATING FEASIBLE MAPS

By JONATHAN DÖNSZELMANN AND JAAP DE JONG

For experiments in this paper, *MAPFM* instances (sometimes called maps) are randomly generated. To generate these random maps, we created a program called the Multi-Agent Pathfinding instance generator (*MPIG*) which creates these maps. We designed *MPIG* to always create maps which are feasible. In this appendix, we show how *MPIG* works, and how *MPIG* can guarantee that each map is feasible by providing a generic procedure which can be used to solve any map generated by *MPIG*.

To simplify the explanation, we first demonstrate the process of generating feasible *MAPF* instances.

### A.1. PROPERTIES OF A FEASIBLE *MAPF* INSTANCE

*MAPF* instances are feasible whenever it is possible for every agent to reach their goal. *MPIG* ensures that this is always possible, by guaranteeing that every generated map has the following two properties:

1. Every map is connected. There are no disconnected subgraphs.
2. Maps with  $k$  agents and  $k$  goal vertices contain at least  $k$  vertices with three or more neighbours (i.e. vertices where at least three adjacent vertices are traversable). From now on, these locations will be called *branch vertices*. Branch vertices are important because at these locations, agents can pass each other as can be seen in Figure 7.

To guarantee the first property is satisfied, *MPIG* starts generation of maps at a single location, and neighbours of that vertex are recursively expanded (by either adding obstacles or traversable locations) to generate the rest of the map. Obstacles are not created when this would cause a disconnected subgraph to be created. The second property is guaranteed by simply discarding a map and generating a new map whenever there are fewer than  $k$  branch vertices. Discarding is used because chances are high that random maps contain more than  $k$  branch vertices.

Some maps which do not have these two properties may still be feasible, but this can not be guaranteed by the

proof given in Section A.2.

### A.2. PROOF OF FEASIBILITY

In this section, it is proven that when maps are connected, and there are at least  $k$  branch vertices, they are feasible. This proof consists of the following three parts which will be considered separately:

1. Every agent can always travel to a branch vertex from their starting location
2. When every agent is on a branch vertex, they can move to reorder themselves such that every agent can be on any of the branch vertices.
3. There exists a configuration of agents on branch vertices that allows all agents to go to their goal.

#### Part 1

**Theorem A.1.** *There is always at least one agent which can reach a branch vertex without collisions.*

*Proof of Theorem A.1.* Let  $v$  be a branch vertex and  $a_1$  be the agent with the shortest path to  $v$ . Then,  $a_1$  can reach  $v$  without collisions.  $\square$

**Theorem A.2.** *All agents can reach a branch vertex without collisions.*

*Proof of Theorem A.2.* There are at least  $k$  branch vertices. Thus, there are enough branch vertices to accommodate all agents. The process for every agent to reach this branch vertex is as follows:

Step 1: a single agent moves to a branch vertex, which is possible according to Theorem A.1.

Step 2: an attempt is made to move another agent  $a_i$  to a branch vertex. Two situations may occur:

1. Agent  $a_i$  can move to a branch vertex  $u$  without obstruction.
2. Another agent  $a_j$  which previously moved to a branch vertex  $v$  obstructs  $a_i$  from reaching a branch vertex  $u$ .

In the second situation, agent  $a_j$  can instead move to vertex  $u$ , freeing vertex  $v$  for agent  $a_i$ .

Step 2 can be repeated until all agents reach a branch vertex thus proving Theorem A.2.  $\square$

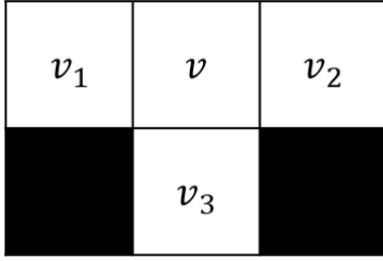


Figure 8: The trivial map with a single branch vertex

## Part 2

**Theorem A.3.** *In every map, from every neighbour of a branch vertex  $u$ , there exists a path to a non-branch vertex that does not traverse  $u$ .*

*Proof of Theorem A.3.* A proof by construction follows:

In the trivial map with a single branch vertex (see figure 8), Theorem A.3 holds, since each neighbour is a non-branch vertex.

Every possible map with at least one branch vertex can be derived from the trivial map by adding more open vertices around it.

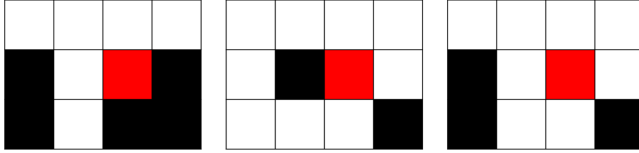


Figure 9: Three different ways of connecting vertices. Red vertices are added to maps.

Adding a new vertex  $u$  can have one of three effects on each neighbour  $v$ :

1.  $v$  has a single neighbour. Connecting to  $v$  makes  $v$  a two-neighbour vertex. Theorem A.3 trivially holds for  $v$ , because  $v$  is not a branch vertex.
2.  $v$  has two neighbours  $v_1$  and  $v_2$ . Connecting to  $v$  makes  $v$  a three-neighbour vertex, i.e. a branch vertex. If  $v_1$  or  $v_2$  have fewer than three neighbours, then Theorem A.3 trivially holds. If  $v_1$  or  $v_2$  is a branch vertex, then they must already be part of the map and Theorem A.3 holds for  $v_1$  and  $v_2$ . Since  $u$  is a branch vertex, it is possible to pathfind to one of the remaining neighbours of  $u$ , which are directly or indirectly connected to a non-branch vertex. Since Theorem A.3 holds for all neighbours  $v_1, v_2$  and  $u$  of  $v$ , it must now also hold for  $v$ .
3.  $v$  has three neighbours,  $v_1, v_2$  and  $v_3$ . Connecting to  $v$  makes  $v$  a four-neighbour vertex. The same reasoning used in effect 2 can be used to show that Theorem A.3 still holds for  $v$ .

Adding a new vertex  $u$  can also have one of the following effects on  $u$  itself:

1. It can create a new two-neighbour vertex  $u$  by connecting two vertices (shown figure 9). Since

$u$  has fewer than three neighbours, Theorem A.3 still holds for  $u$ .

2. It can create a new three-neighbour vertex  $u$  by connecting three vertices (shown figure 9). A vertex with which a connection is made (called  $v$ ), can be in one of three possible configurations for which Theorem A.3 holds, as explained in the previous part of the proof. Since Theorem A.3 holds for all neighbours of  $u$ , it also holds for  $u$  itself since every neighbour is always directly or indirectly connected to a non-branch vertex.
3. It can create a new four-neighbour vertex  $u$  by connecting four vertices (shown in figure 9). A vertex with which a connection is made (called  $v$ ), can be in one of three possible configurations for which Theorem A.3 holds. The reasoning from the previous effect can be used to show that Theorem A.3 also holds in this effect.

For the trivial map from Figure 8, Theorem A.3 trivially holds. Every map with one or more branch vertices can be constructed from the trivial map by adding vertices to it. Adding vertices to a map for which Theorem A.3 holds, was shown to exclusively create new maps for which the Theorem still holds. If a map cannot be derived from the trivial map, then it does not contain branch vertices. Theorem A.3 trivially holds for maps without any branch vertices.

Therefore, Theorem A.3 holds for all generated maps.  $\square$

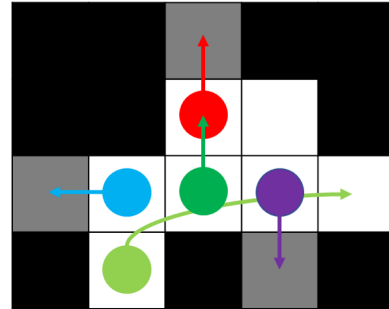


Figure 10: An example of how an agent can pass other agents even if there is no space between branch vertices

Consider the scenario where each agent is positioned on a branch vertex. As a result of theorem A.3, each neighbour of a branch vertex  $u$  has a so-called *diversion vertex*, which is the non-branch vertex that is reachable from the neighbour without visiting  $u$ .

**Theorem A.4.** *An agent  $a_i$  on a branch vertex  $v$  can always be passed by another agent  $a_j$*

*Proof of Theorem A.4.* The branch vertex  $v$  has three neighbours  $v_1, v_2$  and  $v_3$  (as shown in Figure 8).  $a_j$  passing  $v$  means that it is coming from one of the neighbours of  $v$  (say  $v_1$ ) and needs to travel to another one of the neighbours (say  $v_2$ ). For  $a_j$  to travel from  $v_1$  to  $v_2$ ,  $a_i$  must move out of the way to  $v_3$ .  $v_3$  can either be:

- A non-branch vertex. It is therefore empty because all agents are on branch vertices.  $a_i$  can simply move to  $v_3$  and let  $a_j$  pass.
- A branch vertex. In this case, there may be an agent  $a_k$  on  $v_3$ . If there is an agent on  $v_3$ , it must also move out of the way. Theorem A.3 shows that it is always possible to pathfind to a non-branch vertex from neighbours of branch vertices. Since non-branch vertices are empty, this provides a place for agents to move in to make room for passing agents. Therefore,  $a_k$  must move either onto an empty vertex, or move onto a vertex with another agent which after possible repetitions will always find an empty *diversion vertex* to move onto. Figure 10 shows how all agents move out of the way to *diversion vertices* to allow the lime agent to pass.

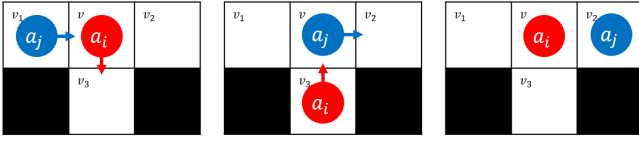


Figure 11: An example of how an agent can pass another agent.

After having encountered one of these two scenarios, agent  $a_j$  has moved to  $v$ , and  $a_i$  has moved out of the way to  $v_3$ . For agent  $a_j$  to now completely pass  $a_i$ ,  $a_j$  must continue to  $v_3$  (these steps are shown in Figure 11). However, if  $v_3$  is another branch vertex, another agent  $a_k$  may be on it. Two situations can now occur:

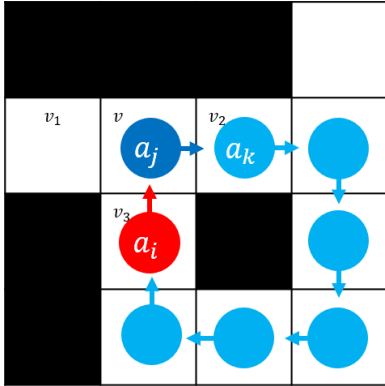


Figure 12: An example of how agents can pass with a single diversion vertex.

- $a_k$  can move out of the way just like  $a_j$  did. Theorem A.3 shows that this is always possible to find a diversion vertex.  $a_i$  can now also move back to  $v$ .
- $a_k$  can not move out of the way. Even though Theorem A.3 shows that there is always a diversion vertex to move out of the way,  $a_j$  moving out of the way may have taken up this diversion vertex. However, if both  $v_2$  and  $v_3$  have the same diversion vertex, a connection must exist between  $v_2$  and  $v_3$ . Because the definition of *MAPF* allows following, it is now possible for  $a_k$  to move out of the way, following agents in front of  $a_k$  in a chain.

The head of the chain is  $a_i$ .  $a_i$  moves back to  $v$ , in a way making  $v$  the diversion vertex. This motion can be seen in Figure 12

After this process,  $a_i$  is back on  $v$  and  $a_j$  has passed to  $v_3$   $\square$

**Theorem A.5.** *Any two agents on adjacent branch vertices (i.e. directly connected or connected with a corridor) can swap places, both moving to the branch vertex where the other agent was standing.*

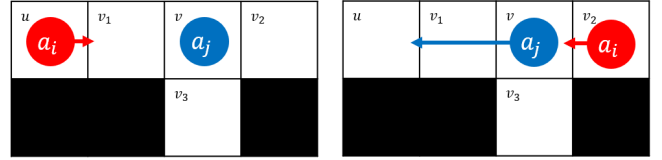


Figure 13: An illustration of two agents swapping by passing each other.

*Proof of Theorem A.5.*

**Lemma A.6.** *The swapping of two agents  $a_i$  and  $a_j$ , positioned on branch vertices  $u$  and  $v$  respectively, is equivalent to  $a_i$  passing  $a_j$  (or vice versa). After the passing, both agents can move to the vertex where the other agent used to be without conflict.*

Theorem A.4 shows that an agent coming from one neighbour of a branch vertex can always pass the branch vertex to move to another neighbour of the branch vertex. Agents  $a_i$  and  $a_j$  can swap by one of the agents passing the other agent on its branch vertex and both agents moving back to the swapped branch vertices without collision. This process can be seen in Figure 13. There is always enough space for an agent to pass another agent because of the diversion squares.

Therefore, two agents on adjacent branch vertices can swap places.  $\square$

**Theorem A.7.** *If all agents are assigned to and located on a branch vertex, they can move to create every other assignment of agents to branch vertices.*

*Proof of Theorem A.7.* Any permutation of a set of elements can be created using only pairwise swaps by using the Steinhaus–Johnson–Trotter algorithm [29].

Not all pairwise swaps are swaps between adjacent elements. However, any pairwise swap of two non-adjacent elements  $a$  and  $b$  can be performed by swapping all the elements between  $a$  and  $b$ . The procedure

The proof of theorem A.5 shows that pairwise swaps of agents on adjacent branch vertices are possible on any map.  $\square$

### Part 3

**Theorem A.8.** *Every connected map with  $n$  agents on  $n$  branch vertices is directly solvable from at least one assignment  $p$  of agents to branch vertices.*

*Proof of Theorem A.8.* Consider the scenario where every agent is positioned on its corresponding goal. By theorem A.2, the agents can all travel to branch squares without collision. This results in a assignment  $p$  of agents to branch squares.  $\square$

**Theorem A.9.** *Every connected map with  $n$  agents and at least  $n$  branch vertices is feasible.*

*Proof of Theorem A.9.* By Theorem A.2, it is possible for every agent to reach a branch vertex. By Theorem A.7, every assignment of agents to branch vertices can

be created. Theorem A.8 shows that there is always an assignment for which the map is solvable. Therefore, every map with at least  $n$  branch vertices is feasible.  $\square$

### A.3. SOLVING MAPFM INSTANCES

**Theorem A.10.** *Every connected MAPFM map with  $n$  agents and at least  $n$  branch vertices is feasible.*

*Proof of Theorem A.10.* A MAPFM instance can be decomposed into many MAPF instances by considering all possible assignments of agents to goals, which can exhaustively be searched. Theorem A.9 shows that every MAPF instance is feasible. As a result, every possible assignment of agents to goals of a MAPFM instance is also feasible. Therefore, all MAPFM instances with  $n$  agents and  $n$  branch vertices are feasible as well.  $\square$